Interval Scheduling

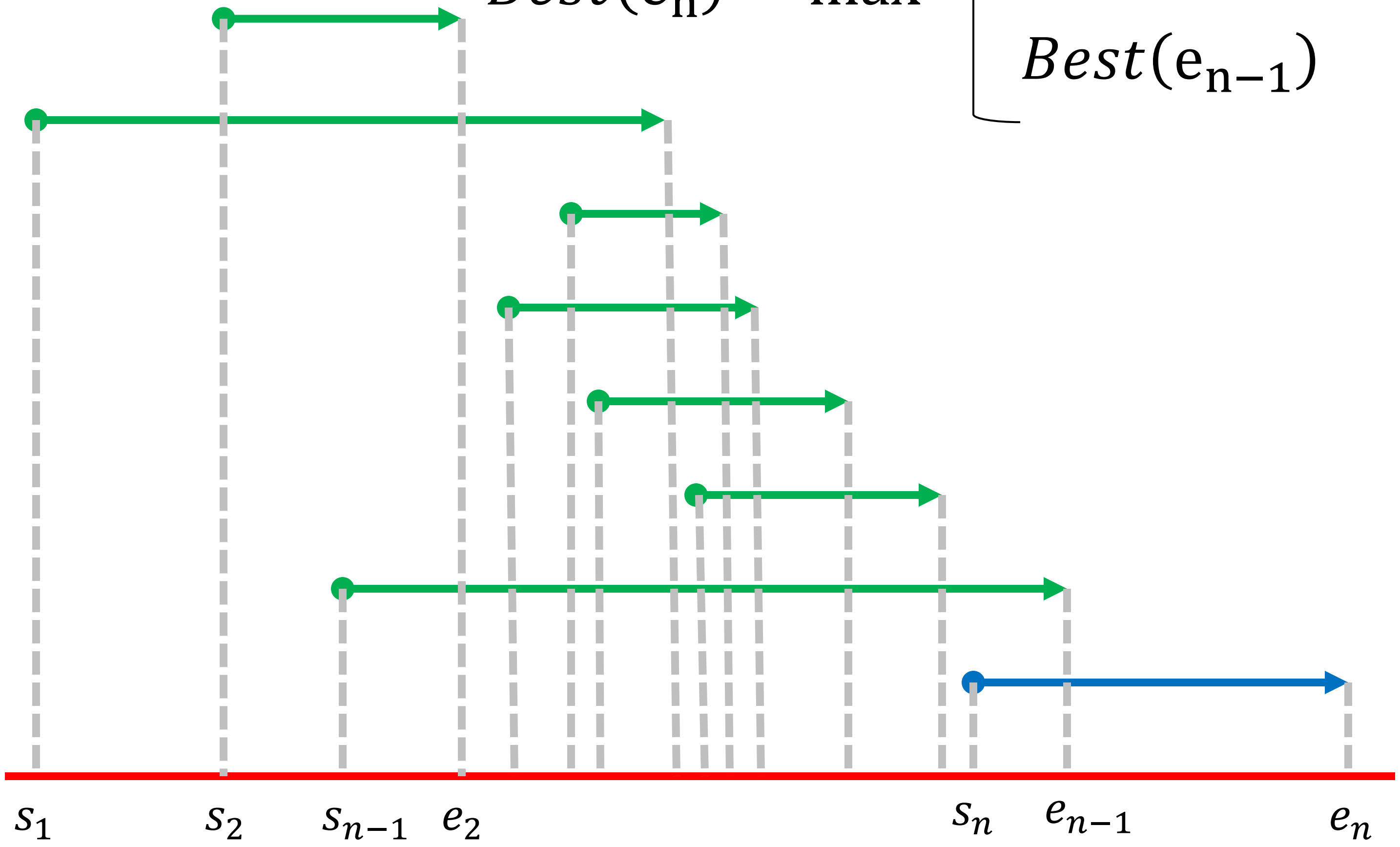
- Input: List of events with their start and end times (sorted by end time)
- Output: largest set of non-conflicting events (start time of each event is after the end time of all preceding events)

[1, 2.25]	Alumni Lunch
[2, 3.25]	CS4102
[3, 4]	CHS Prom
[4, 5.25]	Bingo
[4.5, 6]	SCUBA lessons
[5, 7.5]	Roller Derby Bout
[7.75, 11]	UVA Football watch party

Interval Scheduling DP

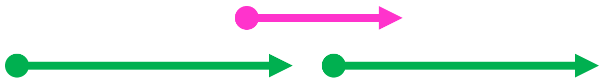
$Best(t)$ = max # events that can be scheduled before time t

$$Best(\mathrm{e_n}) = \max \begin{cases} Best(\mathrm{s_n}) + 1 & \text{Include event } n \\ Best(\mathrm{e_{n-1}}) & \text{Exclude event } n \end{cases}$$

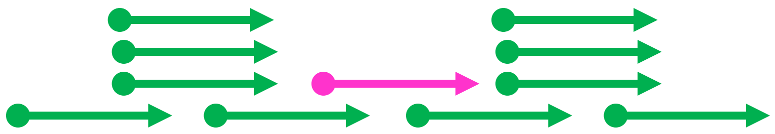


Greedy Interval Scheduling

- Step 1: Identify a greedy choice property
  - Options:
    - Shortest interval



    - Fewest conflicts



    - Earliest start

    - Earliest end

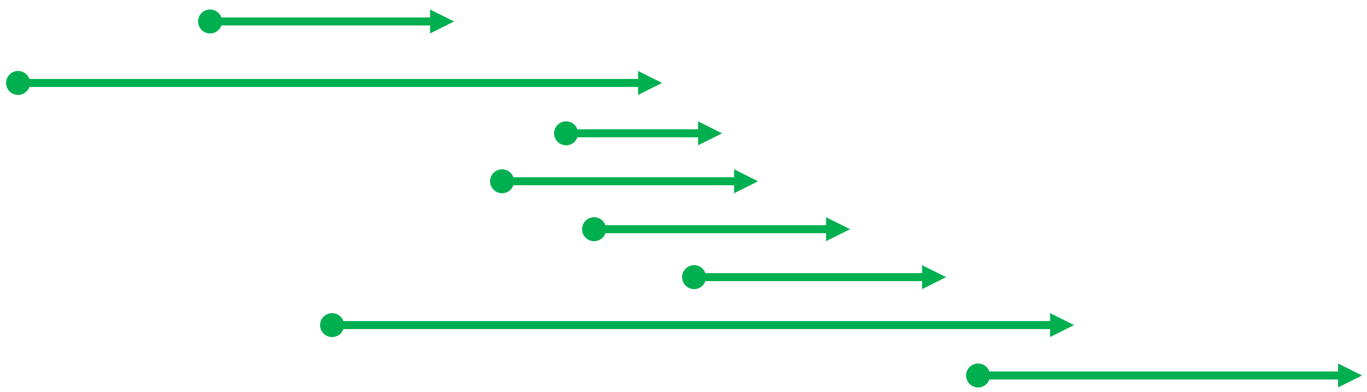
Prove using Exchange Argument

Interval Scheduling Algorithm

Find event ending earliest, add to solution,

Remove it and all conflicting events,

Repeat until all events removed, return solution



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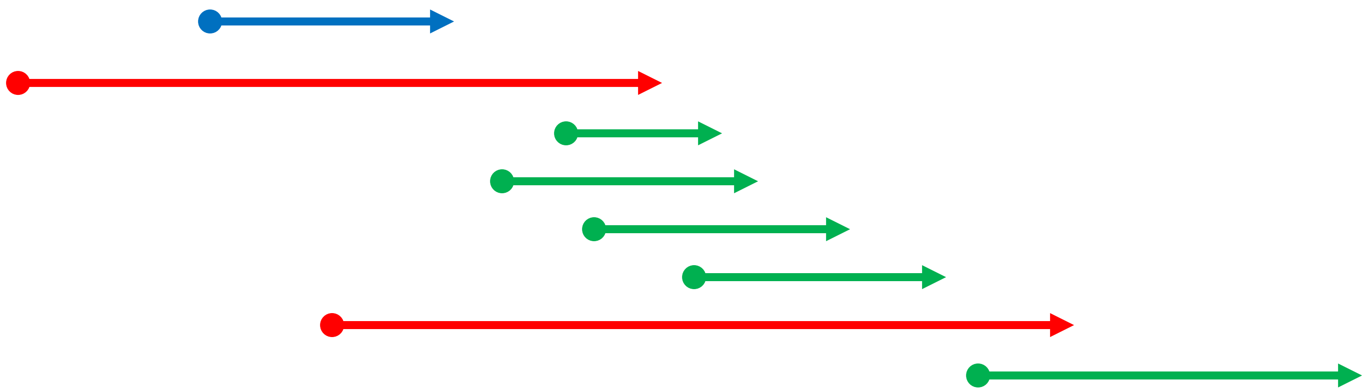

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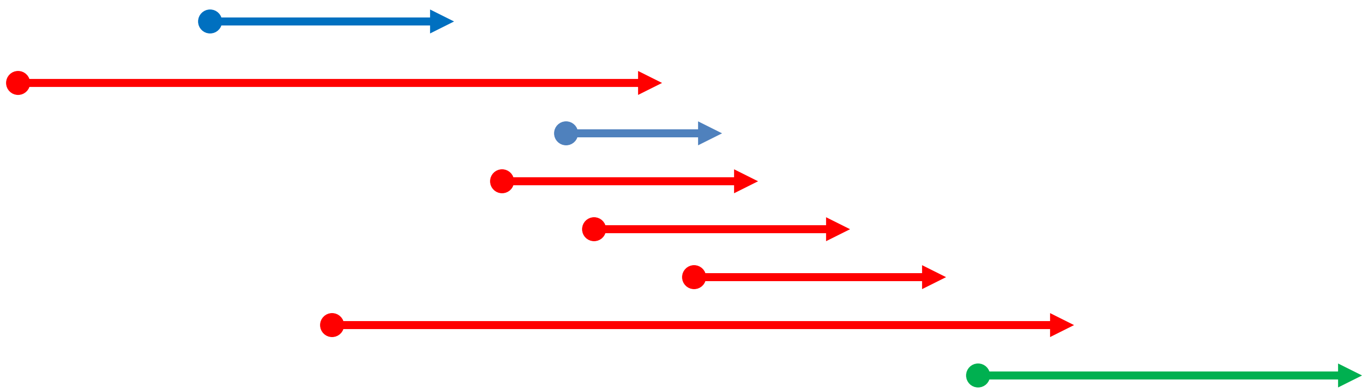

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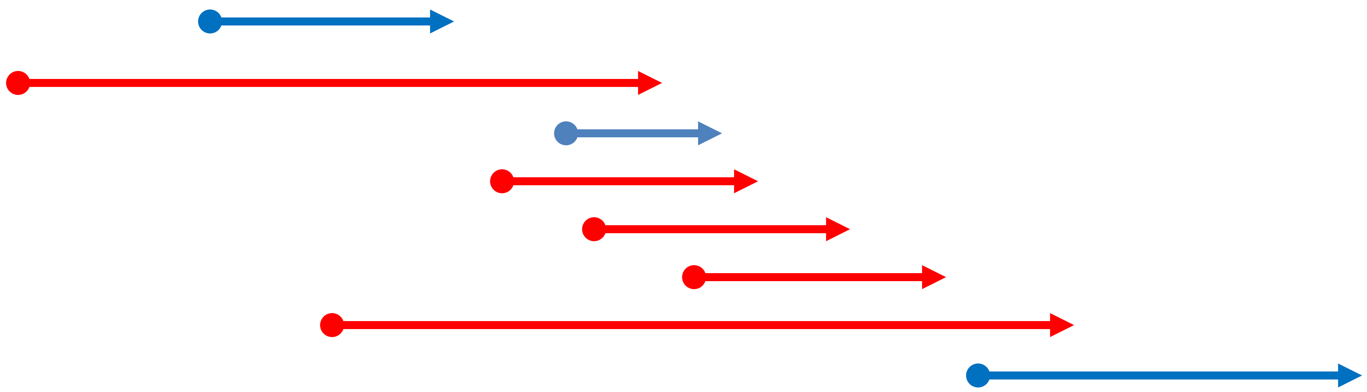

Interval Scheduling Run Time

Find event ending earliest, add to solution,

Remove it and all conflicting events,

Repeat until all events removed, return solution

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Equivalent way

StartTime = 0
For each interval (in order of finish time):                     O(n)
        if begin of interval < StartTime or end of interval < StartTime:     O(1)
                do nothing
        else:
                add interval to solution                          O(1)
                StartTime = end of interval
```

Exchange argument

- Shows correctness of a greedy algorithm
- Idea:
  – Show exchanging an item from an arbitrary optimal solution with your greedy choice makes the new solution no worse
  – How to show my sandwich is at least as good as yours:
    - Show: “I can remove any item from your sandwich, and it would be no worse by replacing it with the same item from my sandwich”

Exchange Argument for Earliest End Time

- Claim: earliest ending interval is always part of <u>some</u> optimal solution

- Let $OPT_{i,j}$ be an optimal solution for time range $[i, j]$
- Let a^* be the first interval in $[i, j]$ to finish overall
- If $a^* \in OPT_{i,j}$ then claim holds
- Else if $a^* \notin OPT_{i,j}$, let a be the first interval to end in $OPT_{i,j}$
  - By definition a^* ends before a, and therefore does not conflict with any other events in $OPT_{i,j}$
  - Therefore $OPT_{i,j} - \{a\} + \{a^*\}$ is also an optimal solution
  - Thus claim holds